Teachers Teaching with Technology™
Professional Development from Texas Instruments

Exploring Common Core Topics in High School Mathematics with the TI-84 Plus Family

Materials for Workshop Instructor*

Materials

- 30 TI-84 Plus CE graphing calculators
- 30 unit-to-unit transfer cables
- 30 standard-A to mini-B USB cables
- Ball of string
- Scissors
- Rulers, meter sticks and tape measures
- Stopwatch/timer
- Small bouncing ball.
- Rubber bands

TI–84 Plus CE Apps

- Cabri Junior – *CabriJr*
- Probability Simulation – *ProbSim*
- Transformation Graphing – *Transfrm*
- Inequality Graphing – *Inequalz*

Notes

Apps can be downloaded from education.ti.com but are included with the instructor files. Apps are not interchangeable between the TI-84 Plus CE, TI-84 Plus C Silver Edition, and TI-84 Plus Silver Edition graphing calculators.

Optional

The workshop includes optional activities that use the TI Connect™ CE and TI-SmartView™ CE software. Determine with the organizer whether computers with the TI Connect™ CE and TI-SmartView™ CE software will be available.

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Introduction to the TI-84 Plus CE Graphing Calculator

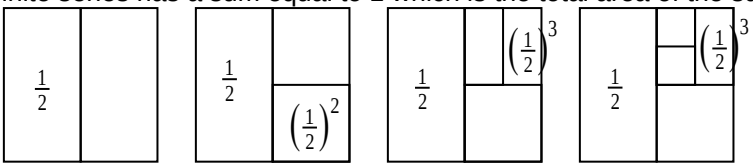
PD Objectives

- This activity provides an introduction to some of the basic keys of the calculator and illustrates possible uses of the home screen for exploration and pattern building.

Materials Needed/Set Up Requirements

Main Focus – Suggested Questions/Strategies for Accomplishing Objectives

- Increase familiarity with the keypad, scrolling history, menus, division key ÷ vs. (−) key, etc.
- Use the stacked fraction template for more than computation, but to build patterns, make conjectures, and motivate students to use algebra to show a conjecture is true. You could break participants into groups to each explore #5 through #11.
- For #7, the sum of powers of $\frac{1}{2}$ can be represented with a 1x1 square which is repeatedly bisected. The infinite series has a sum equal to 1 which is the total area of the square.



- For #11, once you have $\frac{1}{x+1}+\frac{1}{x}\cdot\frac{1}{x+1}$ on the home screen, in the Y= menu position your cursor at Y1 and use 2nd [ENTRY] to "beam it up". Use a friendly window (ZFrac1/10 is nice) and TRACE to show it is equivalent to $y = 1/x$ with a hole at $x = -1$.
- For #12, seeing the table and graph of the two logarithmic expressions may illuminate that one is an exponential function $y = 2^x$ and the other is a linear function $y = 2x$. If using the TI-84 Plus C Silver Edition, you may want to explore with a split screen.

Technology Tips

- When using some Math Templates like mixed number, summation, or log base, it is tempting to press the UP arrow key but resist. Use the RIGHT arrow key, as guided.
- Pressing 2nd LEFT arrow takes you to the beginning of the entry line; 2nd RIGHT arrow takes you to the end of the entry line.
- If appropriate, discuss danger of using mode settings inappropriately, i.e., set Float to 0 (round to nearest integer) and find $\sqrt{2}$. You will get what you asked for– garbage! Or use Mode to set Answers to FRAC (find fractional approximation) and find $5\sqrt{3}$. This is not a calculator error, but what software engineers call GIGO (*Garbage In, Garbage Out*).
- Discuss features of the two pages of the MODE screen.
- Show the 2nd [MEM] 7, 2, 2 reset to reset the defaults on MODE, WINDOW, [FORMAT]…

Summary Reflection Questions

- Discuss appropriate use of the calculator. Now that it can be used to add stacked fractions (or logs of different bases or other features), what can we ask students that we could not ask before?

The Common Core Mathematical Practices – Investigations

PD Objectives

- In these investigations, participants will look at how the TI-84 Plus family can be used to implement the Common Core Mathematical Practices.
- In the process, participants will learn new TI-84 Plus features as well as review old ones.

Materials Needed/Set Up Requirements

Main Focus – Suggested Questions/Strategies for Accomplishing Objectives

- This section and the prior activity should take up the morning of Day 1.
  - **Investigation 1** makes use of the pre-set Zoom windows. Using MP 7 (*Look for and make use of structure*) and MP 8 (*Look for and express regularity in repeated reasoning*), one can generalize the pattern on the home screen to $\frac{1}{x} + \frac{1}{x} \cdot \frac{1}{x+1}$ and use algebraic reasoning to show it is equivalent to $\frac{1}{x}$ for all $x \neq 0, -1$.
  - **Investigation 2** makes use of the scrolling history and summation command (optional). Once participants build the sequence $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{31}{32}, \ldots$, they might predict the *n*th denominator is 2^n and its numerator is 1 less than the denominator, i.e., the *n*th term is $\frac{2^n - 1}{2^n}$.

    Alternatively, they may observe if the previous term is $\frac{a}{b}$, the next term is $\frac{a+b}{2b}$.

    So if $\frac{a}{b} = \frac{2^n - 1}{2^n}$, then the next term is $\frac{a+b}{2b} = \frac{(2^n - 1) + 2^n}{2 \cdot 2^n} = \frac{2 \cdot 2^n - 1}{2 \cdot 2^n} = \frac{2^{n+1} - 1}{2^{n+1}}$

    The figure in Part 1c can illustrate $\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots = \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k = 1$.

    Using MP 2 (*Reason abstractly and quantitatively)*, an argument could be made that the term $\frac{2^n - 1}{2^n} = 1 - \frac{1}{2^n} \to 1$ as $n \to \infty$. Similarly, the formula for a geometric series gives

    $\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots + \left(\frac{1}{2}\right)^n = a\frac{1 - r^n}{1 - r}$ for $a = \frac{1}{2}$ and $r = \frac{1}{2}$, or $\frac{1}{2} \cdot \frac{1 - \left(\frac{1}{2}\right)^n}{1 - \frac{1}{2}} = \frac{1 - \left(\frac{1}{2}\right)^n}{2 \cdot \frac{1}{2}}$

    $= 1 - \left(\frac{1}{2}\right)^n$ or $1 - \frac{1}{2^n}$ as above. It works well to assign different groups parts 2a through 2e.
  - **Investigation 3** builds the sequence $\frac{2}{1}, \frac{5}{2}, \frac{10}{3}, \frac{17}{4}, \ldots$ providing an opportunity to make conjectures for the next numerator. Note that 2 and 5 are 3 apart, 5 and 10 are 5 apart, 10 and 17 are 7 apart, and so on. Some may subtract 1 from each numerator to see they are perfect squares, and in general the *n*th numerator is $n^2 + 1$. Changing the mode to **Un/d** reveals that the mixed number $X\frac{1}{x} = \frac{x^2+1}{x}$, meeting MP7 by making use of structure.

Main Focus – Suggested Questions/Strategies for Accomplishing Objectives

- **Investigation 4** is similar to the previous investigation, where an expression equivalent to $1+\frac{x+1}{x}$ can be shown to be equivalent to $\frac{2x+1}{x}=2+\frac{1}{x}$. Changing the mode to **Un/d** reveals that the expression is the mixed number $2\frac{1}{x}$, again meeting MP7 by making use of structure.
- **Investigation 5** provides an opportunity for modeling (MP4), as well as working with lists and the sequence wizard. The amount *S* on day *n* is $S = 1+2+2^2+2^3+...+2^{n-1}=2^n-1$ which first exceeds 100,000 on Day 17. For groups that finish early, ask them to consider what would happen if the amount tripled, quadrupled, or quintupled.
  For tripling: $S = 1+3+3^2+3^3+...+3^{n-1}=\frac{1}{2}(3^n-1)$
  For quadrupling: $S = 1+4+4^2+4^3+...+4^{n-1}=\frac{1}{3}(4^n-1)$
  For quintupling: $S = 1+5+5^2+5^3+...+5^{n-1}=\frac{1}{4}(5^n-1)$

  The following common core state standards can arise in this investigation (★=Modeling):

  - A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
  - A-SSE.4 Derive the formula for the sum of a finite geometric series (when *r* is not 1), and use the formula to solve problems.★
  - A-CED★.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.★
  - F-BF.1 Write a function that describes a relationship between two quantities. ★
    a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
    b. Combine standard function types using arithmetic operations.
    c. Write a function that describes a relationship between two quantities.
  - F-BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★
  - F-LE★.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

- In **Investigation 6**, the equation is explored for different values of *a* and *b.* Once it is seen that the parameter *a* is "invariant," you could complete a table like the one below.

b	$y = a^{\frac{b}{\log_x a}}$
0	
1	
2	
3	
-1	
1/2	

$\Rightarrow$

b	$y = a^{\frac{b}{\log_x a}}$
0	$y = 1$ (Note: $y = a^0 = 1$)
1	$y = x$
2	$y = x^2$
3	$y = x^3$
-1	$y = 1/x$
1/2	$y = \sqrt{x}$

This may lead to the conjecture that, in general, this is the power function $y = x^b$.

Taking logarithms to the base *x* of both sides of the equation $y = a^{\frac{b}{\log_x a}}$ reveals that $\log_x y = \log_x a^{\frac{b}{\log_x a}}$, so $\log_x y = \frac{b}{\log_x a} \cdot \log_x a$. Therefore $\log_x y = b$. Writing this in exponential form gives $y = x^b$, which explains why changing the parameter *a* to any positive number greater than 1 results in no change in the expression.

- In **Investigation 7**, the function $y = \log_x 10$ is explored for values of *x* which are powers of 10 to provide an opportunity to conjecture that the outputs are reciprocals of the powers of 10. Using algebraic reasoning, we can let $y = \log_x 10$. If we write it in exponential form $x^y = 10$ and then take common logarithms of both sides of the equation we obtain $\log x^y = \log 10$. Thus $\log x^y = 1$. Using properties of logs we have $y \log x = 1$. Solving for *y* gives us $y = \frac{1}{\log x}$.

  This matches the conjecture, since for values of $x = 10^p$, the expression simplifies to $y = 1/p$. Alternatively, write the expression $y = \log_x 10$ in exponential form $x^y = 10$, then if $x = 10^p$ is a power of 10, we have $x^y = 10^{py} = 10^1$, so $py = 1$ and $y = 1/p$.

- Depending on the level and expertise of the participants, you might not be able to finish all of the investigations. They can be revisited later in the workshop or left as 'homework'.
- Point out the multiple representation possibilities when working on an investigation (tables, graphs, calculations, equations).
- Many of the investigations have extensions that can be used for differentiation in the workshop.

Technology Tips

- Highlight the use of the shortcut keys ALPHA [F1] – ALPHA [F4]

Summary Reflection Questions

- Which Mathematical Practices are highlighted in each activity?

One-Variable Statistics

PD Objectives

- This activity provides an opportunity to collect and analyze one-variable data.

Materials Needed/Set Up Requirements

- A 3 oz cup for each team of 4 participants and a large 1 lb. bag of M&M's® (if using ideas below).
- If doing the human boxplot, you need string and meter sticks.

Main Focus – Suggested Questions/Strategies for Accomplishing Objectives

- Sample data is provided but this activity works better if participants collect their *own* data.
- The last page collects letters in one's first name. Alternatively, you can deviate from the handout and do the following activity with a cup. (This activity is intentionally not put into the handout to prevent spoilers and to facilitate inquiry-based learning.)

  1. Put participants in teams of 4. Mark a line on each cup, all at the same level. Ask each team to estimate how many M&M's will fill their cup up to the marked line. Once guesses are submitted, fill cups to the marked line. Display each team's guess and actual amount. Put teams' guesses in L1 and actual in L2.
  2. Ask participants to predict how a boxplot of guesses would differ than a boxplot of the actual. (There will likely be more variation and possible outliers for the guesses.) Use the TI-84 to make a boxplot of guesses and a boxplot of actual on the same screen. Compare.
  3. Display the boxplot and histogram for guesses, then the boxplot and histogram for actual.
  4. Make a scatter plot of the (guess, actual) data using the largest mark and overlay the line $y = x$. Discuss what each data point represents. Ask:

     - What is true about a data point if it falls above the line $y = x$? Actual > Guess (underestimate)
     - What is true about a data point if it falls below the line $y = x$? Guess > Actual (overestimate)
     - What is true about a data point if it falls right on the line $y = x$? The team nailed it.

       (Reward these teams with the rest of the bag of candy.)

Technology Tips

- For the scatterplot, participants may prefer to make a scatter plot of (actual, guess) data instead of (guess, actual) data. Just change Xlist to Actual (L2) and Ylist to Guess (L1).
- Try to "break" a histogram by assigning Xscl an impossible value.

Summary Reflection Questions

- How can you match a histogram to a box plot? Look at the median.
  See http://higheredbcs.wiley.com/legacy/college/mann/0470444665/applets/applet_01_v4.html
- How is a histogram different than a bar chart?
  See http://www.shodor.org/interactivate/discussions/HistogramsVsBarGraph/

Exploring the Coordinate Plane

PD Objectives

- Take advantage of pre-set windows from the Zoom menu.
- Explore the coordinate plane by graphing a square using a line plot
- Investigate reflections, translations, and dilations of a figure by modifying the coordinates.

Materials Needed/Set Up Requirements

Main Focus – Suggested Questions/Strategies for Accomplishing Objectives

- Which pre-set windows from the Zoom menu are square? Friendly? Both?
- Discuss how some preset zoom windows are named by the increment that the trace cursor moves on a graph with each press of the left or right arrow key.
- The Chart has (-*x*,-*y*) twice. This is deliberate. One should describe as reflection and the other as rotation.
- Discuss the extensions that are included. Ask participants if they can come up with any other ideas.

Technology Tips

- The shortcut 2nd [MEM], then 2:Defaults…, then 2:Reset. This turns StatPlots off as well as the functions in Y=. It does not clear the Y=.
- If you want to make a line graph of a closed figure, you should end with the same point that you started with.
- The TI Connect software could be used to capture students' artwork for printing.

Summary Reflection Questions

- How does the window variable Xres affect graphing?

  Answer: Xres sets pixel resolution (1 through 8) for function graphs only. The default is 1. At Xres=1, functions are evaluated and graphed at each pixel on the *x*-axis. Singularities (holes and asymptotes) are checked. A graph such as $y = \tan x$ in connected mode would not show "false asymptotes." Graphing is slower but at the best resolution. If Xres is 2 to 8, singularities are not checked, false asymptotes would appear, and graphing is faster. For example, if Xres=8 functions are evaluated and graphed at every eighth pixel along the *x*-axis.

Pass the Ball

PD Objectives

- Participants will measure the time it takes to complete a specific set of tasks as a group.
- After developing a mathematical model of each pattern, participants will make predictions about how long it will take to repeat the same task a certain number of times.

Materials Needed/Set Up Requirements

- TI-84 Plus or TI-84 Plus C Silver Edition graphing calculator
- Stopwatch/timer
- Small bouncing ball (tennis/racquet balls work well)

Main Focus – Suggested Questions/Strategies for Accomplishing Objectives

- Collect data and create a scatterplot to model.
- Students will use slope to come up with their initial model
- Manual-Fit and LinReg are done as part of the extension.
- Discussion notes and a student worksheet are included.
- Another extension to this activity involves the concept of residuals and addresses CCSS S-ID. A nice activity called “Square it Up!” (ID: 11409) can be found at the activities exchange and is included in the three day materials.

Technology Tips

- Instead of using Zoom 9:ZoomStat, have students think about the window settings.
- Use the graph and table to answer prediction questions.

Summary Reflection Questions

- Is there an advantage of using one regression method over another?

Online Resources

PD Objectives

- Explore resources available at education.ti.com.
- Browse for classroom activities at the All Activities, TI Math, and the Common Core State Standards webpages.
- Search for activities using the Standards Search and Textbook Search.
- Explore information regarding professional development.

Materials Needed/Set Up Requirements

- Computers with internet connection

Main Focus – Suggested Questions/Strategies for Accomplishing Objectives

- Visit *education.ti.com* to explore the available resources.
- This is a self-paced activity. Move around and answer questions about the website.
- Have teachers share any interesting findings.

Technology Tips

- Point out the Downloads tab, where teachers can get the latest OS, apps and guidebooks.
- If there is time available and TI Connect™ and/or TI-SmartView™ software is loaded on the computers, teachers may want to look at some of the features of the software by investigating the information in the Appendices.

Summary Reflection Questions

- How might you use the education.ti.com web site in your planning to address the Common Core State Standards?